Learning math and problem solving in context

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Is a math teacher in Tokyo Gakugei University International Secondary School [TGUISS] in Tokyo, Japan, and has been teaching math at the secondary level for more than 15 years, and at TGUISS for 6 years now. TGUISS is very research-oriented because it's attached to Tokyo Gakugei University which is a national university, and has strong teacher educational programs, maintaining many attached schools and playing a national role in the development of educational policy and innovations in teacher education. It has been offering the Middle Years Programme of International Baccalaureate for grade 7-10 since 9 years ago, and is the first public school to offer this program in Japan. Then, it has started offering Diploma Programme for grade 11 and 12 since this April. Furthermore, 1/3 of our students returned from overseas or had experienced secondary school abroad. With this school circumstance, she currently has been teaching some high school mathematics subjects and the DP mathematics Standard Level in English.

Since I attended the CASIO Global Teachers Meeting 2015 in last summer, I have kept thinking how I can tell our unique Japanese math education and its characteristics to foreign math teachers and even Japanese people who are not in the educational field because these are merely recognized by them unless they are educators. So it's my great pleasure to tell about Japanese math education to the teachers, people or researchers in the educational field in Germany. I will talk about the characteristics of teaching style and our math education in my school.

The biggest characteristic on Japanese math education compared to the one in other countries is that it is very important to foster and develop students' deep understanding of mathematical concepts. This is related to one of the reasons that why calculators are not used for studying the basic mathematical contents in Japan. Another characteristic on Japanese math education is to use the learning and teaching style of well-structured problem solving in class, especially at elementary level. This was, for example, revealed and supported in the "Teaching Gap" published in 1999, which summarized the result of research by American researchers Stigler and Hielbert. The Teaching Gap gave a huge impact on mathematics researchers and teachers in the United States. It became one of very famous and must-read educational books in the U.S. and even in Japan. Although these researches and statements are old, this characteristic of problem solving style lesson still have been maintained even nowadays.

On the other hand, it is pity for me to mention that this problem solving style lesson is less taken place in junior high schools than elementary schools. As it comes to high schools, there are only a few high school teachers who are fond of this teaching style. Because of this, many Japanese people think that math classes are like explaining the contents, and giving lots of exercises by the time they graduate their high schools.

However, our school teachers believes mathematical problem solving style is very essential to reach the goal for our math education throughout 6 years of secondary level, which is to foster

mathematical literacies. Mathematical literacies we believe are such as the ability to recall and use their knowledge of mathematics, concepts and skills in a variety of contexts, and the ability to use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems. Therefore, we often use the investigation tasks we created which always contain the problems in a context. In fact, we created 4 original textbooks for from 7th grade to 10th grade. I show an example of them below.

This is the investigation task from the 10th original textbook for the introduction of studying exponential functions until defining the expression for exponential functions.

Investigation 1 Is it safe to drink an unfinished beverage from a plastic bottle?

Compact size plastic bottles are now widely used. Therefore, we often keep unfinished drinks for a long time and carry them around because they can be capped. However, is it safe to drink it after keeping it for a long time?

Once you drink something from a plastic bottle putting your mouth on it, bacteria residing in your own mouth will pass to it. Depending on the surrounding conditions, the number of bacteria may start increasing inside the bottle.

One of the books on microbiology says:

> Microorganisms grow very fast if the surrounding environment such as the temperature, humidity, nutrition, and oxygen are suitable for them.
> Growth rate (division rate) of bacteria can be shown as the time it takes for one cell to become two cells after the start of a cell division, which is called generation time (G). ・・・(omitted)
> The value of this generation time becomes smaller as the growth rate becomes faster.
> The generation time (G) of E. coli and lactobacillus is normally 20 minutes, however, it becomes around 15 minutes in a very suitable growth media.
>
> From *Applied Microbiology for nutritionists*, revised and written by Amo Mikio and Koishikawa Ninji, published by Koseikan

If you drink something that contains lots of bacteria, you may suffer from food poisoning. Suppose that the growth rate (division rate) is one hour. Let's study how the number of bacteria increases and consider if it is safe to drink it.

■**Q1** Steven had a sip of green tea from a plastic bottle at eight o'clock in the morning, and put the bottle into his bag. Later, at noon, he checked the number of bacteria that can cause food poisoning, and he found that there were 240 bacteria per 1 mL of the tea. How many bacteria do you suppose there will be per 1 mL of the tea one and three hours later?

■**Q2** Calculate each number of bacteria per 1 mL for 1 hour later, 2 hours later, 3 hours later・・・, and 8 hours later.

■**Q3** Make a graph for showing the number of bacteria, and plot the numbers you obtained in Q2.

■**Q4** These bacteria are said to cause food poisoning if the number of them exceeds 100,000 per 1 mL.

What time will it be?

■**Q5** Based on the result of **Q2**, if the number of bacteria x hours later is $N(x)$, each of $N(1)$,$N(2)$, $N(3)$, $N(4)$ can be shown as follows:

$N(1)=240\times2$ $N(2)=240\times2^2$ $N(3)=240\times2^3$ $N(4)=240\times2^4$

Following the above, create an equation to determine the number of bacteria x hours later "$N(x)$".

The expected number of bacteria "$N(x)$" can be one number if the value of x is determined. Therefore, $N(x)=240\times2^x$ is a function of x.

Exponential functions

When $a\neq0$, $b\neq1$, and $b>0$, the function,

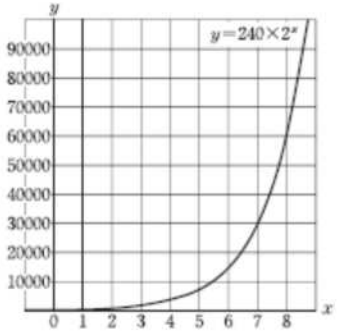
$y=a\times b^x$, (a is non "0" constant)

is called an **exponential function** of x to the **base** b.

This investigation is continued to the introduction of learning logarithmic functions as follows:

Investigation 2 Reconsider the case we studied in the Investigation 1 "Is it safe to drink an unfinished beverage from a plastic bottle?"

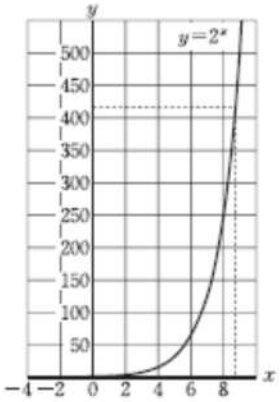
In the Investigation 1 "Is it safe to drink an unfinished beverage from a plastic bottle?", we found the approximate time when the chance of having food poisoning is high. In this section, let's try to find a more accurate time.



■**Q1** Supposing that the time when the number of bacteria exceeds 100,000 is x hours from noon, create an equation of x to figure out the situation.

■**Q2** With your calculator, find the value of x.

■**Q3** Supposing that the time when the number of bacteria exceeds 100,000 is x hours from noon, create an equation of x to figure out the situation.



The solution to the equation created in **Q1** is the answer to the question how many times should 2 be multiplied to be equal to $\frac{100000}{240}$ (= $\frac{1250}{3}$). And, as we can guess from the answer to **Q2**, this value can be an infinite decimal, which is an infinite sequence of integers.

Therefore, this value can be shown as follows: $\log_2\frac{1250}{3}$.

When $ax=y$, the number x is called the **logarithm** of y to the **base** a, which can be shown as $x=\log_a y$. Also, the number y is called the **antilogarithm** of the logarithm of x to the base a.

***Students would use their graphics calculators: fx-CG20 throughout these investigations, and for learning**

graphs of exponential and logarithmic functions more. Incidentally, in my school, all the students have their own calculators from their 1st grade.

Lastly, I would like to mention about our next new curriculum. Japanese national curriculum significantly effects on entire elementary and secondary educations nationwide. Since past 3 years, the Ministry of Education, Culture, Sports, Science and Technology recently has planned and announced its outline step by step for the new curriculum which will be implemented in 2018. One of its huge impacts for the next curriculum is that the National Center Test for University Admission will be changed to a new test system. Especially, mathematics tests would be changed from solving normal theoretical math problems to solve the problems in a real life situation to evaluate how students could apply their mathematical knowledge and skills to solve in context. Moreover, the way of answering tests would be changed from mark sheet to written. This would drive the high school teachers to change the type of mathematical problems and tasks they bring into their classes, and their teaching styles.

By this educational reform in the future, I am hoping that the quality of Japanese math education will be improved, and Japanese secondary students will enjoy and find the meaningful purposes to learn mathematics more and obtain the higher quality of mathematical knowledge, skills and thinking. Incidentally, in my school, our recent goal is to create the original textbooks for the 11th and 12th grade. With this educational change, we are also hoping that the math education in our school will be the cutting edge as a pioneer school.

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References

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